First Semester Examination
2017/2018 Academic Session

January 2018

EAS661 – Advanced Structural Mechanics

Duration : 2 hours

Please check that this examination paper consists of EIGHT (8) pages of printed material before you begin the examination.

Instructions: This paper contains **SIX (6)** questions. Answer **FOUR (4)** questions.

All questions must be answered in English.

Each question **MUST BE** answered on a new page.

1. (a) Using suitable specialization of basic equations in 3D elasticity, derive the governing differential equation for the linear elastic bar problem shown in **Figure 1** where E : elastic modulus, A : cross-sectional area, w : uniformly distributed load per unit length. Next, using the derived governing differential equation, obtain the expression of axial displacement u and axial stress σ_x for the linear elastic bar problem. Both ends of the bars are fixed.

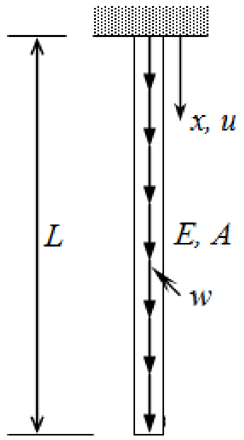

Figure 1

[15 marks]

(b) **Figure 2** shows a thin plate structure subjected to in-plane (x-y plane) loads on the boundary.The thin plate is also subjected to in-plane body force $R_x(x,y)$ and $R_y(x,y)$ in x and y direction, respectively.

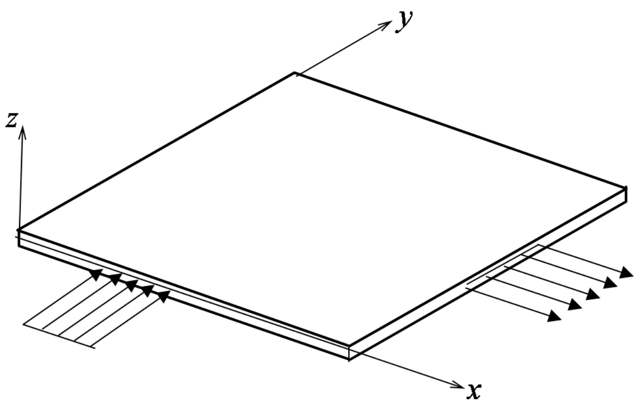

Figure 2

(i) Using a diagram of an infinitesimal volume taken from the interior of the thin plate in **Figure 2**, show clearly the non-zero stress components.

(ii) Next, using the diagram shown in (i), derive the equilibrium equation in x-direction.

[10 marks]

2. (a) The strain energy U_p stored in an elastic body can be expressed using the following equation:

$$U_P = \int_{vol} v_p d(vol)$$

where v_p is strain energy density. Using the above equation of U_p, derive the equation of strain energy stored in the linear elastic prismatic bar shown in **Figure 3**, where E: elastic modulus, A: cross-sectional area, L: length of the bar and e: elongation of the bar.

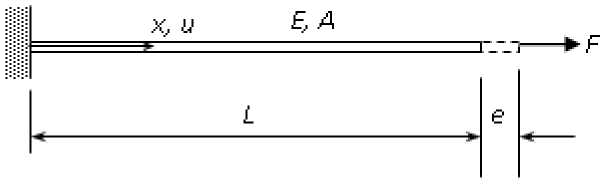

Figure 3

[5 marks]

(b) **Figure 4** shows a cantilever beam with length l subjected to a uniformly distributed load of intensity w and a vertical point load P at a distance of $\frac{2}{3}l$ from the fixed support. One linear spring with spring constant k is attached to the free end. The following expression for lateral displacement field v has been suggested:

$$v = C_0\left(1 - \cos\frac{\pi x}{2l}\right)$$

where C_0 is a constant. Show that the above displacement field is kinematically admissible. Next, solve for the constant C_0 by using Rayleigh-Ritz method and principle of minimum potential energy. Flexural rigidity of the column is EI.

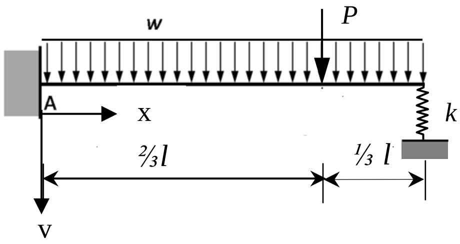

Figure 4

[20 marks]

3. **Figure 5** shows a stepped bar which is fixed at both ends. Length of both portions 1-2 and 2-3 is 0.5*L.* Elastic modulus of the bar is *E* and cross-sectional area of portion 1-2 and 2-3 is *A* and 1.5*A*, respectively. The bar is subjected to a uniformly distributed load *w* and 1.5*w* per unit length for portion 1-2 and 2-3, respectively. Using piece-wise Rayleigh-Ritz method, obtain the axial displacement at point 2. Divide the bar into two portions 1-2 and 2-3 and assume linear displacement field for each portion.

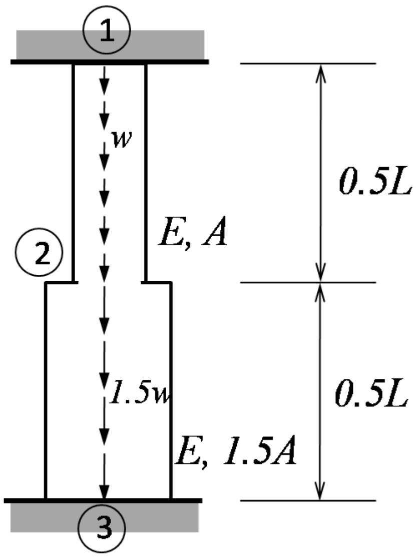

Figure 5

[25 marks]

4. (a) Write down the element stiffness matrices and global matrix for the three bars assembly which is loaded with force P, and constrained at the two ends in terms of E, A and L as shown in **Figure 6a**.

[5 marks]

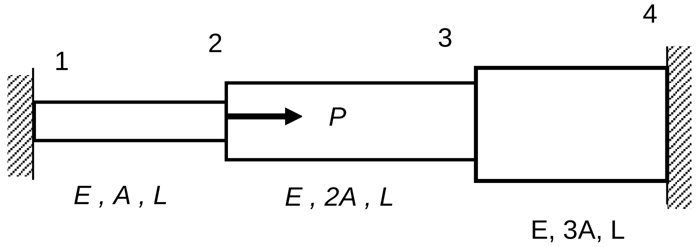

Figure 6a

(b) Write down the element stiffness matrices and global matrix for the two bars assembly which is loaded with force 10P at node 2 as shown in **Figure 6b**. The bar assembly is constrained at end B and free at end A with a gap of Δ at end A. Given the value of P = 60 kN, E = 20 kN/mm^2, L = 200 mm, A= 250 mm^2 and Δ = 1.2 mm, determine:

(i) the displacement at nodes 1, 2 and 3

(ii) the support reaction forces at A

[20 marks]

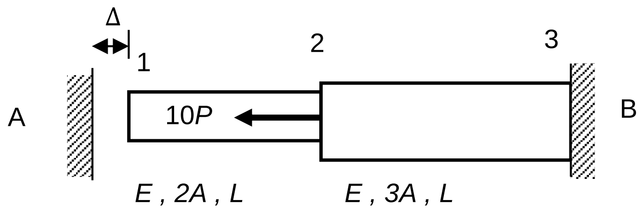

Figure 6b

5. (a) Two plates shown in **Figure 7a** and **7b**, shall be analysed as a plane strain problem. Both plates are divided into 9 elements. Each node has been labelled accordingly. Calculate the bandwidth, B = (R+1) NDOF for the plate assuming two degrees of freedom at each node.

[5 marks]

(b) Rearrange the node labeling in such a way that a minimum value of R is obtained.

[5 marks]

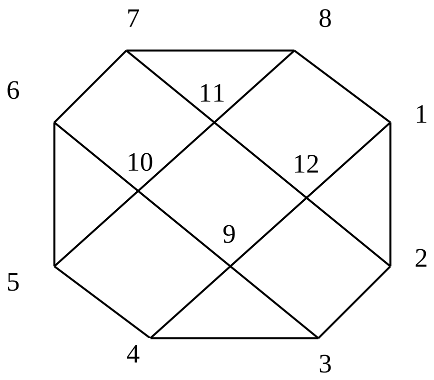

Figure 7a

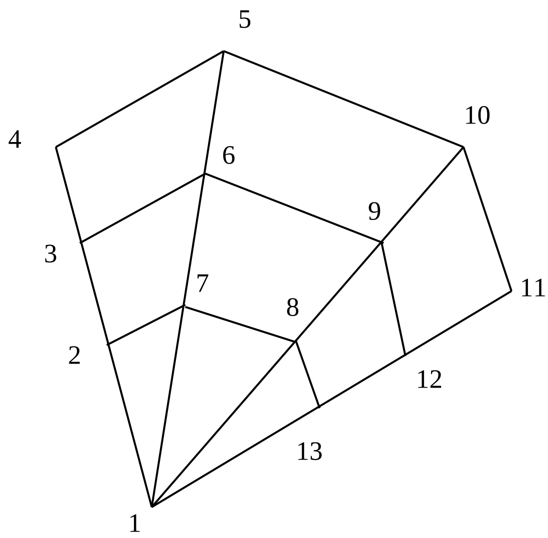

Figure 7b

(c) **Figure 7c** shows a frame structure labeled as nodes 1, 2 and 3, subjected to a nodal forces of P = 20 kN at node 2 and uniformly distributed load of 6 kN/m. The frame is fixed at node 1 and pinned at node 3. Given the value of E = 207 GPa, I = 3×10^{-5} m^4 and A = $0.005 m^2$.

(i) Derive the global stiffness matrix for the frame.

(ii) Determine the deflection u_2 , v_2 , θ_2 in unit metre and rad, respectively.

[15 marks]

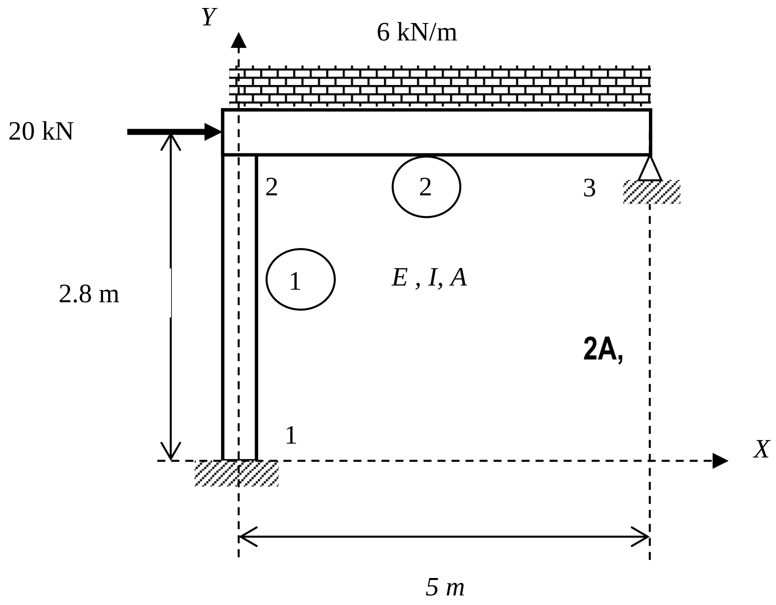

Figure 7c

Given the stiffness of the beam element in dimensional space :

$$
\begin{array}{cc}
 & \begin{array}{cccc} v_i & \theta_i & v_j & \theta_j \end{array} \\
k = \dfrac{EI}{L^3} & \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}
\end{array}
$$
for the beam element

$$
\begin{array}{cc}
 & \begin{array}{cc} u_i & u_j \end{array} \\
k = & \begin{bmatrix} k & -k \\ -k & k \end{bmatrix}
\end{array}
$$
for the spring element

6. (a) Explain the importance of model validity and accuracy of the following factors in the modeling procedures for Finite Element Method.

(i) geometry

(ii) material properties

(iii) loading conditions

[9 marks]

...8/-

(b) Show clearly in a step by step manner the development process of a stiffness matrix, $[K]^e$, for a triangular element in a state of plane stress as shown in **Figure 8**. Next calculate the displacements at node 3 and node 2. Given $E = 10^3$ kN/m^2, $\nu = 0.3$ and t = 0.5 m.

[16 marks]

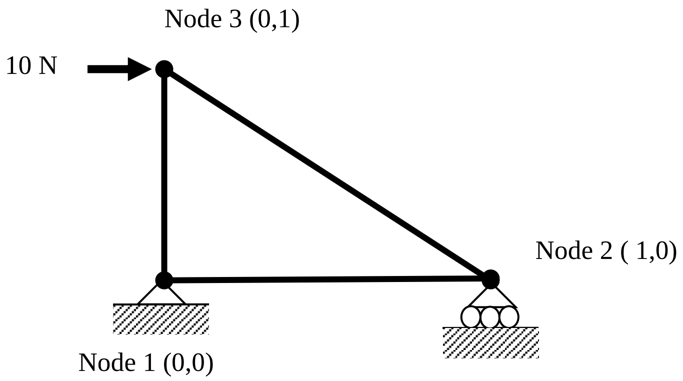

Figure 8

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